

Transition from GCSE to A Level

Maths skills

1 Numbers and units

1.1 Units and prefixes

A key criterion for success in biological maths lies in the use of correct units and the management of numbers. The units scientists use are from the *Système Internationale* – the SI units. In biology, the most commonly used SI base units are metre (m), kilogram (kg), second (s), and mole (mol). Biologists also use SI derived units, such as square metre (m²), cubic metre (m³), degree Celsius (°C), and litre (l).

To accommodate the huge range of dimensions in our measurements they may be further modified using appropriate prefixes. For example, one thousandth of a second is a millisecond (ms). Some of these prefixes are illustrated in the table below.

| Multiplication factor | Prefix | Symbol |
|-----------------------|--------|--------|
| 10 ⁹ | giga | G |
| 10 ⁶ | mega | M |
| 10 ³ | kilo | k |
| 10 ⁻² | centi | c |
| 10 ⁻³ | milli | m |
| 10 ⁻⁶ | micro | μ |
| 10 ⁻⁹ | nano | n |

Practice questions

- A burger contains 4 500 000 J of energy. Write this in:
a kilojoules b megajoules.
- HIV is a virus with a diameter of between 9.0×10^{-8} m and 1.20×10^{-7} m.
Write this range in nanometres.

1.2 Powers and indices

Ten squared = $10 \times 10 = 100$ and can be written as 10^2 . This is also called 'ten to the power of 2'.

Ten cubed is 'ten to the power of three' and can be written as $10^3 = 1000$.

The power is also called the index.

Fractions have negative indices:

one tenth = $10^{-1} = 1/10 = 0.1$

one hundredth = $10^{-2} = 1/100 = 0.01$

Any number to the power of 0 is equal to 1, for example, $29^0 = 1$.

If the index is 1, the value is unchanged, for example, $17^1 = 17$.

When multiplying powers of ten, you must *add* the indices.

So $100 \times 1000 = 100\,000$ is the same as $10^2 \times 10^3 = 10^{2+3} = 10^5$

When dividing powers of ten, you must *subtract* the indices.

So $100/1000 = 1/10 = 10^{-1}$ is the same as $10^2/10^3 = 10^{2-3} = 10^{-1}$

But you can only do this when the numbers with the indices are the same.

So $10^2 \times 2^3 = 100 \times 8 = 800$

And you can't do this when adding or subtracting.

$10^2 + 10^3 = 100 + 1000 = 1100$

$10^2 - 10^3 = 100 - 1000 = -900$

Remember: You can only add and subtract the indices when you are multiplying or dividing the numbers, not adding or subtracting them.

Practice questions

3 Calculate the following values. Give your answers using indices.

a $10^8 \times 10^3$

b $10^7 \times 10^2 \times 10^3$

c $10^3 + 10^3$

d $10^2 - 10^{-2}$

4 Calculate the following values. Give your answers with and without using indices.

a $10^5 \div 10^4$

b $10^3 \div 10^6$

c $10^2 \div 10^{-4}$

d $100^2 \div 10^2$

1.3 Converting units

When doing calculations, it is important to express your answer using sensible numbers. For example, an answer of $6230\ \mu\text{m}$ would have been more meaningful expressed as $6.2\ \text{mm}$.

If you convert between units and round numbers properly, it allows quoted measurements to be understood within the scale of the observations.

To convert $488\,889\ \text{m}$ into km :

A kilo is 10^3 so you need to divide by this number, or move the decimal point three places to the left.

$488\,889 \div 10^3 = 488.889\ \text{km}$

However, suppose you are converting from mm to km : you need to go from 10^3 to 10^{-3} , or move the decimal point six places to the left.

$333\ \text{mm}$ is $0.000\,333\ \text{km}$

Alternatively, if you want to convert from $333\ \text{mm}$ to nm , you would have to go from 10^{-9} to 10^{-3} , or move the decimal point six places to the right.

$333\ \text{mm}$ is $333\,000\,000\ \text{nm}$

Practice question

5 Calculate the following conversions:

a $0.004\ \text{m}$ into mm

b $130\,000\ \text{ms}$ into s

c $31.3\ \text{ml}$ into μl

d $104\ \text{ng}$ into mg

6 Give the following values in a different unit so they make more sense to the reader.

Choose the final units yourself. (Hint: make the final number as close in magnitude to zero as you can. For example, you would convert 1000 m into 1 km.)

- a 0.000 057 m b 8 600 000 μl c 68 000 ms d 0.009 cm

2 Decimals, standard form, and significant figures

2.1 Decimal numbers

A decimal number has a decimal point. Each figure *before* the point is a whole number, and the figures *after* the point represent fractions.

The number of decimal places is the number of figures *after* the decimal point. For example, the number 47.38 has 2 decimal places, and 47.380 is the same number to 3 decimal places.

In science, you must write your answer to a sensible number of decimal places.

Practice questions

- 1 New antibiotics are being tested. A student calculates the area of clear zones in Petri dishes in which the antibiotics have been used. List these in order from smallest to largest.

0.0214 cm² 0.03 cm² 0.0218 cm² 0.034 cm²

- 2 A student measures the heights of a number of different plants. List these in order from smallest to largest.

22.003 cm 22.25 cm 12.901 cm 12.03 cm 22 cm

2.2 Standard form

Sometimes biologists need to work with numbers that are very small, such as dimensions of organelles, or very large, such as populations of bacteria. In such cases, the use of scientific notation or standard form is very useful, because it allows the numbers to be written easily.

Standard form is expressing numbers in powers of ten, for example, 1.5×10^7 microorganisms.

Look at this worked example. The number of cells in the human body is approximately 37 200 000 000 000. To write this in standard form, follow these steps:

Step 1: Write down the smallest number between 1 and 10 that can be derived from the number to be converted. In this case it would be 3.72

Step 2: Write the number of times the decimal place will have to shift to expand this to the original number as powers of ten. On paper this can be done by hopping the decimal over each number like this:

6.3900000000

until the end of the number is reached.

In this example that requires 13 shifts, so the standard form should be written as 3.72×10^{13} .

For very small numbers the same rules apply, except that the decimal point has to hop backwards. For example, 0.000 000 45 would be written as 4.5×10^{-7} .

Practice questions

- 3 Change the following values to standard form.

a 3060 kJ b 140 000 kg c 0.000 18 m d 0.000 004 m

4 Give the following numbers in standard form.

a 100

b 10 000

c 0.01

d 21 000 000

5 Give the following as decimals.

a 10^6

b 4.7×10^9

c 1.2×10^{12}

d 7.96×10^{-4}

2.3 Significant figures

When you use a calculator to work out a numerical answer, you know that this often results in a large number of decimal places and, in most cases, the final few digits are 'not significant'. It is important to record your data and your answers to calculations to a reasonable number of significant figures. Too many and your answer is claiming an accuracy that it does not have, too few and you are not showing the precision and care required in scientific analysis.

Numbers to 3 significant figures (3 s.f.):

7.88 25.4 741

Bigger and smaller numbers with 3 significant figures:

0.000 147 0.0147 0.245 39 400 96 200 000 (notice that the zeros before the figures and after the figures are *not* significant – they just show you how large the number is by the position of the decimal point).

Numbers to 3 significant figures where the zeros *are* significant:

207 4050 1.01 (any zeros between the other significant figures *are* significant).

Standard form numbers with 3 significant figures:

9.42×10^{-5} 1.56×10^8

If the value you wanted to write to 3 s.f. was 590, then to show the zero was significant you would have to write:

590 (to 3 s.f.) or 5.90×10^2

Remember: For calculations, use the same number of figures as the data in the question with the lowest number of significant figures. It is not possible for the answer to be more accurate than the data in the question.

Practice question

6 Write the following numbers to **i** 2 s.f. and **ii** 3 s.f.

a 7644 g

b 27.54 m

c 4.3333 g

d $5.995 \times 10^2 \text{ cm}^3$

7 The average mass of oxygen produced by an oak tree is 11800 g per year.

Give this mass in standard form and quote your answer to 2 significant figures.

3 Working with formulae

It is often necessary to use a mathematical formula to calculate quantities. You may be tested on your ability to substitute numbers into formulae or to rearrange formulae to find specific values.

3.1 Substituting into formulae

Think about the data you are given in the question. Write down the equation and then think about how to get the data to substitute into the equation. Look at this worked example.

A cheek cell has a 0.06 mm diameter. Under a microscope it has a diameter 12 mm. What is the magnification?

$$\text{magnification} = \text{image size (mm)} \div \text{object size (mm)} \quad \text{or} \quad M = \frac{I}{O}$$

Substitute the values and calculate the answer:

$$M = 12 \text{ mm} / 0.06 \text{ mm} = 12 / 0.06 = 200$$

Answer: magnification = $\times 200$ (magnification has no units)

Sometimes an equation is more complicated and the steps need to be carried out in a certain order to succeed. A general principle applies here, usually known by the mnemonic BIDMAS. This stands for **B**rackets, **I**ndices (functions such as squaring or powers), **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction.

Practice questions

- 1 Calculate the magnification of a hair that has a width of 6.6 mm on a photograph. The hair is 165 μm wide.
- 2 Estimate the area of a leaf by treating it as a triangle with base 2 cm and height 9 cm.
- 3 Estimate the area of a cell by treating it as a circle with a diameter of 0.7 μm . Give your answer in μm^2 .
- 4 An *Amoeba* population starts with 24 cells. Calculate how many *Amoeba* cells would be present in the culture after 7 days if each cell divides once every 20 hours. Use the equation $N_t = N_0 \times 2^n$ where N_t = number after time t , N_0 = initial population, n = number of divisions in the given time t .
- 5 In a quadrat sample, an area was found to contain 96 aphids, 4 ladybirds, 22 grasshoppers, and 3 ground beetles. Calculate the diversity of the site using the equation $D = 1 - \sum \left(\frac{n}{N} \right)^2$ where n = number of each species, N = grand total of all species, and D = diversity.

Remember: In this equation there is a part that needs to be done several times then summed, shown by the symbol Σ .

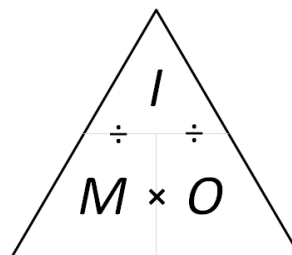
3.2 Rearranging formulae

Sometimes you will need to rearrange an equation to calculate the answer to a question. For example, the relationship between magnification, image size, and actual size of specimens in micrographs usually uses the equation $M = \frac{I}{O}$, where M is magnification, I is size of the image, and O = actual size of the object.

You can use the algebra you have learnt in Maths to rearrange equations, or you can use a triangle like the one shown.

Cover the quantity you want to find. This leaves you with either a fraction or a multiplication:

$$M = I \div O \qquad O = I \div M \qquad I = M \times O$$



Practice questions

- 6 A fat cell is 0.1 mm in diameter. Calculate the size of the diameter seen through a microscope with a magnification of $\times 50$.
- 7 A Petri dish shows a circular colony of bacteria with a cross-sectional area of 5.3 cm^2 . Calculate the radius of this area.
- 8 In a photograph, a red blood cell is 14.5 mm in diameter. The magnification stated on the image is $\times 2000$. Calculate the real diameter of the red blood cell.
- 9 Rearrange the equation $34 = 2a/135 \times 100$ and find the value of a .
- 10 The cardiac output of a patient was found to be $2.5 \text{ dm}^3 \text{ min}^{-1}$ and their heart rate was 77 bpm. Calculate the stroke volume of the patient.
Use the equation: cardiac output = stroke volume \times heart rate.
- 11 In a food chain, efficiency = $\frac{\text{biomass transferred}}{\text{biomass taken in}} \times 100$
A farmer fed 25 kg of grain to his chicken. The chicken gained weight with an efficiency of 0.84. Calculate the weight gained by the chicken.

4 Magnification

To look at small biological specimens you use a microscope to magnify the image that is observed. The microscope was developed in the 17th century. Anton van Leeuwenhoek used a single lens and Robert Hooke used two lenses. The lenses focus light from the specimen onto your retina to produce a magnified virtual image. The magnification at which observations are made depends on the lenses used.

4.1 Calculating the magnifying power of lenses

Lenses each have a magnifying power, defined as the number of times the image is larger than the real object. The magnifying power is written on the lens.

To find the magnification of the virtual image that you are observing, multiply the magnification powers of each lens used. For example, if the eyepiece lens is $\times 10$ and the objective lens is $\times 40$ the total magnification of the virtual image is $10 \times 40 = 400$.

Practice questions

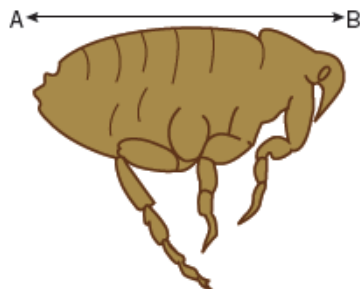
- 1 Calculate the magnification of the virtual image produced by the following combinations of lenses:
- a objective $\times 10$ and eyepiece $\times 12$ b objective $\times 40$ and eyepiece $\times 15$

4.2 Calculating the magnification of images

Drawings and photographs of biological specimens should always have a magnification factor stated. This indicates how much larger or smaller the image is compared with the real specimen.

The magnification is calculated by comparing the sizes of the image and the real specimen. Look at this worked example.

The image shows a flea which is 1.3 mm long. To calculate the magnification of the image, measure the image (or the scale bar if given) on the paper (in this example, the body length as indicated by the line A–B).



For this image, the length of the image is 42 mm and the length of the real specimen is 1.3 mm.

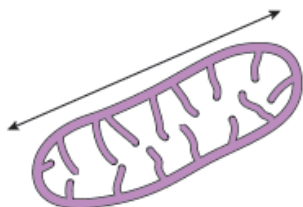
$$\text{magnification} = \frac{\text{length of image}}{\text{length of real specimen}} = 42/1.3 = 32.31$$

The magnification factor should therefore be written as $\times 32.31$

Remember: Use the same units. A common error is to mix units when performing these calculations. Begin each time by converting measurements to the same units for both the real specimen and the image.

Practice question

- 2 Calculate the magnification factor of a mitochondrion that is 1.5 μm long.



4.3 Calculating real dimensions

Magnification factors on images can be used to calculate the actual size of features shown on drawings and photographs of biological specimens. For example, in a photomicrograph of a cell, individual features can be measured if the magnification is stated. Look at this worked example.

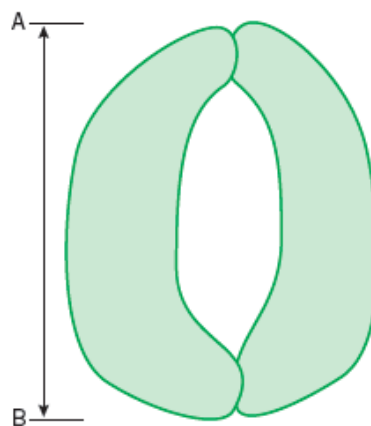
The magnification factor for the image of the open stoma is $\times 5000$.

This can be used to find out the actual size of any part of the cell, for example, the length of one guard cell, measured from A to B.

Step 1: Measure the length of the guard cell as precisely as possible. In this example the image of the guard cell is 52 mm long.

Step 2: Convert this measurement to units appropriate to the image. In this case you should use μm because it is a cell.

So the magnified image is $52 \times 1000 = 52\,000 \mu\text{m}$



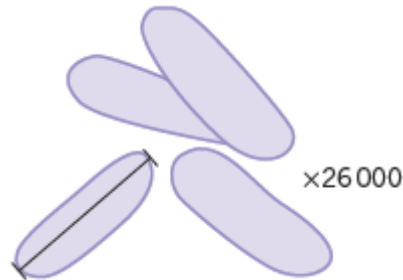
Step 3: Rearrange the magnification equation (see Topic 3.2) to get:

$$\text{real size} = \text{size of image} / \text{magnification} = 52\,000 / 5000 = 10.4$$

So the real length of the guard cell is 10.4 μm .

Practice question

3 Use the magnification factor to determine the actual size of a bacterial cell.



5 Percentages and uncertainty

A percentage is simply a fraction expressed as a decimal. It is important to be able to calculate routinely, but is often incorrectly calculated in exams. These pages should allow you to practise this skill.

5.1 Calculating percentages as proportions

To work out a percentage, you must identify or calculate the total number using the equation:

$$\text{percentage} = \frac{\text{number you want as a percentage of total number}}{\text{total number}} \times 100\%$$

For example, in a population, the number of people who have brown hair was counted.

The results showed that in the total population of 4600 people, 1800 people had brown hair.

The percentage of people with brown hair is found by calculating:

$$\begin{aligned} & \frac{\text{number of people with brown hair}}{\text{total number of people}} \times 100 \\ & = \frac{1800}{4600} \times 100 = 39.1\% \end{aligned}$$

Practice questions

1 The table below shows some data about energy absorbed by a tree in a year and how some of it is transferred.

| | |
|---|-----------------------------|
| Energy absorbed by the tree in a year | 3 600 000 kJ/m ² |
| Energy transferred to primary consumers | 2240 kJ/m ² |
| Energy transferred to secondary consumers | 480 kJ/m ² |

Calculate the percentage of energy absorbed by the tree that is transferred to

a primary consumers **b** secondary consumers.

- 2 One in 17 people in the UK has diabetes.
Calculate the percentage of the UK population that have diabetes.

5.2 Calculating the percentage change

When you work out an increase or a decrease as a percentage change, you must identify, or calculate, the total original amount:

$$\% \text{ increase} = \frac{\text{increase}}{\text{original amount}} \times 100$$

$$\% \text{ decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100$$

Remember: When you calculate a percentage change, use the total *before* the increase or decrease, not the final total.

Practice questions

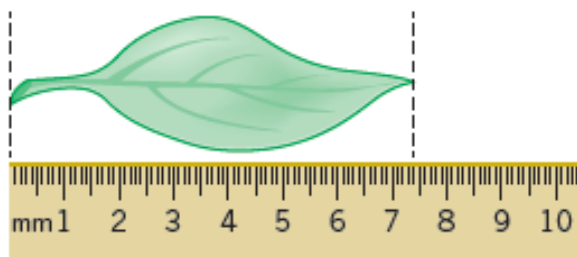
- 3 Convert the following mass changes as percentage changes.

| Sucrose conc. / mol dm ⁻³ | Initial mass / g | Final mass / g | Mass change / g | Percentage change in mass |
|--------------------------------------|------------------|----------------|-----------------|---------------------------|
| 0.9 | 1.79 | 1.06 | | |
| 0.7 | 1.86 | 1.30 | | |
| 0.5 | 1.95 | 1.70 | | |
| 0.3 | 1.63 | 1.76 | | |
| 0.1 | 1.82 | 2.55 | | |

5.3 Measurement uncertainties

When you measure something, there will always be a small difference between the measured value and the true value. This may be because of the size of the scale divisions on your measuring equipment, or the difficulty of taking the measurement. This is called an uncertainty.

To estimate the uncertainty of a measurement with an instrument with a marked scale such as a ruler, a good rule of thumb is to let the uncertainty be equal to half the smallest division on the scale being used.



Using a ruler with a mm scale, the length of the leaf seems to be 74 mm. The smallest division is 1 mm, so the uncertainty is 0.5 mm.

The true length is therefore 74 mm +/- 0.5 mm.

Practice question

- 4 Give the uncertainty for the following pieces of equipment:
- a large measuring cylinder with 2 cm³ divisions
 - b digital stopwatch timer measuring to the nearest hundredth of a second
 - c thermometer with 0.1 °C divisions.

5.4 Calculating percentage uncertainties

The uncertainty is the range of possible error either side of the true value due to the scale being used, so the value recorded for the measurement = closest estimate +/- uncertainty.

The difference between the true value and the maximum or minimum value is called the **absolute error**.

Once the absolute error has been established for a particular measurement, it is possible to express this as a percentage uncertainty or **relative error**. The calculation to use is:

$$\text{relative error} = \frac{\text{absolute error}}{\text{measured value}} \times 100\%$$

In the leaf example above, the absolute error is +/-0.5 mm.

The relative error is therefore:

$$0.5/74 \times 100\% = 0.7\%$$

Practice questions

- 5 Complete the table to show the missing values in the last two columns.

| Measurement made | Equipment used | Absolute error | Relative error |
|---|-------------------------------|----------------|----------------|
| Length of a fluid column in a respirometer is 6 mm | mm scale | 0.5 mm | |
| Volume of a syringe is 12 cm ³ of liquid | 0.5 cm ³ divisions | | |
| Change in mass of 1.6 g | balance with 2 d.p. | | |

6 Scatter graphs and lines of best fit

The purpose of a scatter graph with a line of best fit is to allow visualisation of a trend in a set of data. The graph can be used to make calculations, such as rates, and also to judge the correlation between variables. It is easy to draw such a graph but also quite easy to make simple mistakes.

6.1 Plotting scatter graphs

The rules when plotting graphs are:

- Ensure that the graph occupies the majority of the space available:
 - In exams, this means more than half the space

- Look for the largest number to help you decide the best scale
- The scale should be based on 1, 2, or 5, or multiples of those numbers
- Ensure that the dependent variable that you measured is on the y-axis and the independent variable that you varied is on the x-axis
- Mark axes using a ruler and divide them clearly and equidistantly (i.e. 10, 20, 30, 40 not 10, 15, 20, 30, 45)
- Ensure that both axes have full titles and units are clearly labelled
- Plot the points accurately using sharp pencil 'x' marks so the exact position of the point is obvious
- Draw a neat best fit line, either a smooth curve or a ruled line. It does not have to pass through all the points. Move the ruler around aiming for:
 - as many points as possible on the line
 - the same number of points above and below the line
- If the line starts linear and then curves, be careful not to have a sharp corner where the two lines join. Your curve should be smooth
- Confine your line to the range of the points. Never extrapolate the line beyond the range within which you measured
- Add a clear, concise title.

Remember: Take care, use only pencil and check the positions of your points.

Practice questions

- 1 Use your calculated data in Topic 5.2 question 3 to plot a graph of % mass change against sucrose concentration.
- 2 For each of the tables of data:
 - a Plot a scatter graph
 - b Draw a line of best fit
 - c Describe the correlation

| Turbidity of casein samples at different pH | |
|---|-----------------------------|
| pH | % transmission (blue light) |
| 9.00 | 99 |
| 8.00 | 99 |
| 6.00 | 87 |
| 5.00 | 67 |
| 4.75 | 26 |
| 4.50 | 30 |
| 4.00 | 24 |
| 3.75 | 43 |
| 3.50 | 64 |

| Sodium bicarbonate concentration / % | Rate of oxygen production by pondweed / mm ³ s ⁻¹ |
|--------------------------------------|---|
| 6.5 | 1.6 |
| 5.0 | 2.1 |
| 3.5 | 1.2 |
| 2.0 | 0.8 |
| 1.0 | 0.5 |
| 0.5 | 0.2 |

